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MATHEMATICAL MODELING OF TRANSITION HEAT AND HYDRAULIC PROCESSES IN THE WATER TUBE HEATING SYSTEM WITH RADIATOR THERMOSTATS

The modern building normative base Ukraine provides installation radiator thermostat on heating devices water heating systems. In spite of the declared producers energy-efficient effect in 10 – 20% of the annual cost of heat and comparatively not high cost, the energy saving measure is not widely spread in Ukraine. It is related to number of peculiarities of domestic of systems heating: use of hydraulic elevator thermal units, a small number of buildings equipped with heat counters, the constant hydraulic regime of systems heating and district heating, etc. This work is devoted to investigation of thermal and hydraulic regimes of tube heating systems with thermostatic controls.

For the simulation transient thermal and hydraulic regimes of heating systems with radiator thermostats in this paper proposed of the mathematical model. Its peculiarity is a comprehensive consideration of various factors (both disturbing and regulating) on the thermal and hydraulic modes of work automated heating system. This mathematical model can be used to create computer programs and perform a variety of researches in the field automatic control thermal mode of premises. The research results can be applied to the design and optimization of modern control systems tempering heat.

Keywords: *mathematical model, heating system, thermostatic valve.*

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПЕРЕХІДНИХ ТЕПЛОГІДРАВЛІЧНИХ ПРОЦЕСІВ У ВОДОТРУБНИХ СИСТЕМАХ ОПАЛЕННЯ З РАДІАТОРНИМИ ТЕРМОСТАТАМИ

Розглянуто особливості отримання енергозберігаючого ефекту при застосуванні термостатичних клапанів у системах опалення будівель. Встановлено, що для його збільшення режим роботи термостатів повинен узгоджуватися з тепловим та гідравлічним режимом усієї системи опалення. Роботу присвячено дослідженню теплових та гідравлічних режимів водотрубних систем опалення з термостатичними регуляторами. Розроблено комплексну математичну модель перехідних процесів у приміщеннях із двотрубними водяними системами опалення з термостатами, яка дозволяє розраховувати теплові та гідравлічні режими в автоматизованих системах опалення. Розрахунки виконуються з урахуванням теплофізичних характеристик огороджуючих конструкцій будівлі та попереднього налаштування термостатичного клапана. Доведено, що результати досліджень можуть застосовуватися для проектування й оптимізації сучасних систем керування відпуском теплоти.

Ключові слова: *математична модель, система опалення, термостатичний клапан.*

Introduction. The modern normative basis [1] provides installation radiator thermostats, heating devices for water heating systems. Thermostatic valves developed by different manufacturers for both one- and two-pipe systems for heating. In opinion of manufacturer, this energy-efficient measure should provide heat savings of 10 - 20% of its annual expenditure. This is possible by limiting heat proceeds from the heating system in excess of the set temperature value internal air.

Overview of recent sources of research and publications. There are mathematical models of thermal modes of premises [2, 3], mathematical models of heating systems [4, 5]. There is information on design features, construction and hydraulic characteristics radiator thermostats [6, 7]. However, little data on the dynamics of thermal and hydraulic processes that occur during the work of these regulators in the system heating. This is not to predict work radiator thermostats in different conditions.

Selection not solved earlier of parts the general problem. To correctly describe the thermal hydraulic modes that occur in the heating system when using the radiator thermostat required complex mathematical model. It should take into account the convection and radiation heat flow in the room, transient conditions in the heating system and the work of radiator thermostat.

Problem statement. The aim is to develop a mathematical model to study the thermal and hydraulic modes water-pipe heating system with radiator thermostats.

Basic material and results. Transient thermal conditions of each heat capacity multilayer outer enclosure of the building with sufficient accuracy can describe the thermal conductivity differential equation

$$\frac{\partial t_{3Ki(x,\tau)}}{\partial \tau} = \frac{\lambda_i}{c_i \rho_i} \frac{\partial^2 t_{3Ki(x,\tau)}}{\partial x_i^2} , \tag{1}$$

where t_{3Ki} – temperature i-th layer, °C;

τ – hourly coordinate, s;

λ_i – coefficient of thermal conductivity i-th construction material of wall, W/(m·°C);

c_i – heat capacity i-th layer, J/(kg·°C);

ρ_i – density i-th layer, kg/m^3;

x_i – spatial coordinate i-th layer, kg/m^3.

Boundary conditions at the border contact between two adjacent layers (i та i+1)

$$-\lambda_i \frac{\partial t_{3Ki}}{\partial x_i}(\delta_1, \tau) = \lambda_{(i+1)} \frac{\partial t_{3K(i+1)}}{\partial x_{(i+1)}}(\delta_{(i+1)}, \tau) . \tag{2}$$

Boundary conditions at the inner wall surface (x = 0)

$$-\lambda \frac{\partial t_{3K}}{\partial x}(0, \tau) = \alpha_B^K \left(t_B - t_{3K(0,\tau)}\right) + \alpha_B^R \left(t_R - t_{3K(0,\tau)}\right) , \tag{3}$$

where α_B^K – convective heat transfer coefficient at the inner wall surface, W/(m^2°C);

α_B^R – coefficient of radiation heat transfer at the inner wall surface, W/(m^2°C);

t_B – temperature of internal room air, °C;

t_R – medium radiation temperature of internal room air, °C.

The boundary conditions at the outer wall surface (x = δ)

$$-\lambda \frac{\partial t_{3K}}{\partial x}(\delta, \tau) = \alpha_3 \left(t_{3K(\delta,\tau)} - t_3\right) , \tag{4}$$

where α_3 – heat transfer coefficient at the outer surface of the wall, W/(m^2°C);

t_3 – ambient air temperature, °C.

The difference calculating thermal conditions internal heat capacity structures of the external is only symmetry boundary conditions. Therefore, in the center of this design is no heat flow

$$\lambda_i \frac{\partial t_{BKi}}{\partial x_i}(\delta_i / 2, \tau) = 0 \ . \tag{5}$$

The initial of time conditions provide a linear (stationary) temperature distribution in the thickness walling.

To calculate the temperature on the inner surface of the small-heat capacity walling (windows, exterior doors, etc.) write the equation of their heat balance without considering accumulation of heat

$$\alpha_{\mathrm{B}}^{\mathrm{K}} F_{\mathrm{MK}}(t_B - t_{MK}) + \alpha_{\mathrm{B}}^{R} F_{MK}(t_R - t_{MK}) = \frac{F_{MK}}{R_{MK}}(t_{\mathrm{MK}} - t_3) \ , \tag{6}$$

де $F_{\mathrm{MK}} -$ area of small-heat capacity construction, m^2;

$R_{\mathrm{MK}} -$ thermal resistance design without considering heat transfer at its inner surface, (m^2°C)/W;

$t_{\mathrm{MK}} -$ temperature of on the inner surface, °C.

After the reduction in the area from the equation (6) possible to determine the temperature on the inner surface small-heat capacity construction

$$t_{\mathrm{MK}} = \frac{\alpha_{\mathrm{B}}^{\mathrm{K}} t_{\mathrm{B}} + \alpha_{\mathrm{B}}^{R} t_{\mathrm{R}} + t_3 / R_{MK}}{\alpha_{\mathrm{B}}^{\mathrm{K}} + \alpha_{\mathrm{B}}^{R} + 1/R_{MK}} \ . \tag{7}$$

To determine the temperature of the internal environment of the building, use the heat balance equation

$$\begin{aligned} c_{\Pi} m_{\Pi} \frac{dt_{\mathrm{B}(\tau)}}{d\tau} &= \alpha_2^{\mathrm{K}} F_2 \left(t_{\mathrm{CT}(\tau)} - t_{\mathrm{B}(\tau)}\right) - \sum \alpha_{\mathrm{B}}^{\mathrm{K}} F_{3\mathrm{K}} \left(t_{\mathrm{B}(\tau)} - t_{3\mathrm{K}(0,\tau)}\right) - \\ &- \sum \alpha_{\mathrm{B}}^{\mathrm{K}} F_{\mathrm{BK}} \left(t_{\mathrm{B}(\tau)} - t_{\mathrm{BK}(\delta/2,\tau)}\right) - c_{\Pi} G_{inf} \left(t_{\mathrm{B}(\tau)} - t_{3(\tau)}\right) - \\ &- \sum \alpha_{\mathrm{B}}^{\mathrm{K}} F_{\mathrm{MK}} \left(t_{\mathrm{B}(\tau)} - t_{\mathrm{MK}(\tau)}\right) + Q_{\mathrm{BT}} . \end{aligned} \tag{8}$$

where $c_{\Pi} -$ heat capacity of air, J/(kg·°C);

$m_{\Pi} -$ mass of air or equivalent for the environment, kg;

$\alpha_2^K -$ convective heat transfer coefficient near of heating system elements, W/(m^2°C);

$F_2 -$ heat exchange surface area of heating system, m^2;

$t_{CT} -$ temperature of heat exchange elements surface of heating system, °C;

$G_{inf} -$ loss of infiltrated air, kg/s;

$Q_{BT} -$ Domestic and technological heat flow, W.

Summation in equation (8) indicates adding heat flows from walling with different thermophysical properties.

Heat exchange radiation occurs almost instantly, because equations of thermal balance beam components of heat flow to the internal environment of the heated building is stationary character

$$\begin{aligned} & a_2^{\mathrm{R}} F_2 (t_{\mathrm{CT}} - t_{\mathrm{R}}) - \sum a_{\mathrm{B}}^{\mathrm{R}} F_{3\mathrm{K}} \left(t_{\mathrm{R}} - t_{3\mathrm{K}(0,\tau)}\right) - \sum a_{\mathrm{B}}^{\mathrm{R}} F_{\mathrm{BK}} \left(t_{\mathrm{R}} - t_{\mathrm{BK}(\delta/2,\tau)}\right) - \\ & - \sum a_{\mathrm{B}}^{\mathrm{R}} F_{\mathrm{MK}} (t_{\mathrm{R}} - t_{\mathrm{MK}}) = 0 \ . \end{aligned} \tag{9}$$

From the equation (9) can determine the medium-radiation temperature of heating premises

$$t_{\mathrm{R}} = \frac{\alpha_2^{\mathrm{R}} F_2 t_{\mathrm{CT}} + \sum \alpha_{\mathrm{B}}^{\mathrm{R}} F_{3\mathrm{K}}\, t_{3\mathrm{K}(0,\tau)} + \sum \alpha_{\mathrm{B}}^{\mathrm{R}} F_{\mathrm{BK}} t_{\mathrm{BK}(\delta/2,\tau)} + \sum \alpha_{\mathrm{B}}^{\mathrm{R}} F_{\mathrm{MK}} t_{\mathrm{MK}}}{\alpha_2^{\mathrm{R}} F_2 + \sum \alpha_{\mathrm{B}}^{\mathrm{R}} F_{3\mathrm{K}} + \sum \alpha_{\mathrm{B}}^{\mathrm{R}} F_{\mathrm{BK}} + \sum \alpha_{\mathrm{B}}^{\mathrm{R}} F_{\mathrm{MK}}}\ .$$

To describe transient thermal processes in the heating system will replace its equivalent for heat transfer of heating device. Then design scheme two-pipe of heating system takes the form shown in Figure 1.

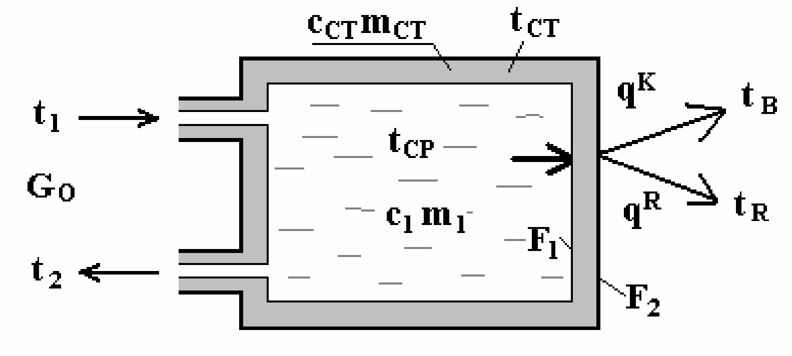

Figure 1 – Calculated scheme "water-pipe" of heating system

In accordance with works [8, 9], possible to use the following mathematical model "water-pipe" of heating system:

$$\begin{cases} c_1 m_1 \dfrac{dt_{2(\tau)}}{d\tau} = \left(c_1 G + \alpha_1 F_1\right)\left(t_{2(\infty)} - t_{2(\tau)}\right) \\ c_1 m_1 \dfrac{dt_{CP(\tau)}}{d\tau} = c_1 G\left(t_1 - t_{2(\tau)}\right) - \alpha_1 F_1\left(t_{CP(\tau)} - t_{CT(\tau)}\right) \\ c_{CT} m_{CT} \dfrac{dt_{CT(\tau)}}{d\tau} = \alpha_1 F_1\left(t_{CP(\tau)} - t_{CT(\tau)}\right) - \alpha_2^K F_2\left(t_{CT(\tau)} - t_{B(\tau)}\right) - \alpha_2^R F_2\left(t_{CT(\tau)} - t_R\right), \end{cases} \tag{10}$$

where $c_1 -$ heat capacity of water, J/(kg·°C);

$m_1 -$ mass of water in the heating system, kg;

$t_2 -$ the coolant temperature in the return pipe of heating system, °C;

$G -$ heat consumption in the heating system, kg/s;

$\alpha_1 -$ heat transfer coefficient between coolant and wall equivalent to the heat transfer heating device, W/(m^2°C);

$F_1 -$ area of internal heat transfer surface, m^2;

$t_{CP} -$ the average temperature of the coolant in the heating system, °C;

$c_{CT} -$ the average heat capacity of heating system elements, J/(kg·°C);

$m_{CT} -$ mass of element of heating system, kg.

To determine the temperature in the return pipe of heating system in stationary conditions possible to apply the formula

$$t_{2(\infty)} = t_{CT} + \left(t_1 - t_{CT}\right)\ell^{-\frac{\alpha_1 F_1}{c_1 G}}\ . \tag{11}$$

Let us assume that each heating device is equipped with an automatic temperature controller direct action. Each thermostat consists of a valve and thermostatic head.

To describe the heating system with radiator thermostats must combine of their thermal and hydraulic mode Fig. 2. The total loss of pressure in the automated heating system will consist of a pressure loss in thermostatic valve and pressure losses in pipes and heating devices

$$\Delta P_{\text{ТВ}} = \Delta P_{КЛ} + \Delta P_{\text{СО}} \ , \tag{12}$$

where $\Delta P_{КЛ} -$ loss of pressure on thermostatic valve, bar;

$\Delta P_{CO} -$ pressure loss in the heating system, bar.

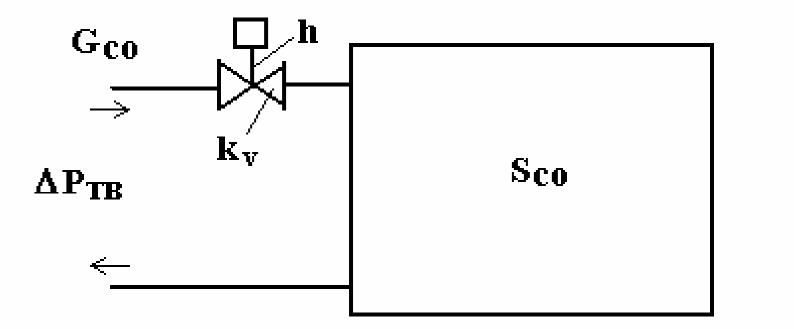

Figure 2 - Design scheme of heating system with individual thermostats

Considering that two-pipe heating system, all valves connected in parallel. Using the method of hydraulic characteristics of equation (12) can be written as

$$\Delta P_{\text{ТВ}} = \left(\frac{1}{(Nk_V)^2} + S_{\text{СО}} \right) G_{CO}^2 \ , \tag{13}$$

where $N -$ the total number of thermostatic valves, pieces;

k_V – coefficient throughput valve, (m^3/h)/bar^{0,5};

S_{CO} – hydraulic characteristics of heating system, bar/(m^3/h)^2;

G_{CO} – heat consumption in the heating system, m^3/h.

From the formula (13) can be heat consumption in the heating system

$$G_{CO} = \sqrt{\frac{\Delta P_{\text{ТВ}}}{\frac{1}{(Nk_V)^2} + S_{\text{СО}}}} \ . \tag{14}$$

The relationship between the move valve stem and his throughput coefficient called costly characteristic of the valve. There are three main types of characteristics: linear, quadratic and equally-percentage. Each type characteristics corresponds their profiling plate valve. For radiator thermostats are used primarily logarithmic characteristics. The coefficient throughput valve with logarithmic characteristic can be defined by the formula

$$k_V = (k_{VB})^h (k_{V3})^{1-h} \ , \tag{15}$$

where k_{VB} – coefficient throughput valve in open position, (m^3/h)/bar^{0,5};

k_{V3} – that, in the closed position, (m^3/h)/bar^{0,5};

$h -$ the relative position of the valve rod.

The coefficient throughput valve in the open state depends on its a standard size, which is determined by hydraulic calculation of heating system. The coefficient throughput valve in the closed position can be taken as being equal to the standard leakage $k_{VП} = 0{,}0005 k_{VB}$.

Hydraulic resistance of the valve is also determined by the provisions of the rod (h). The maximum position of the rod ($h_{max} \le 1$) given at the previous setting valve (installation). Other provisions rod depending on the temperature in the room. Due to this being implemented "negative" an inverse relationship between temperature of air indoors and heat transfer of heaters. Excluding the friction in the valve stem headwork its provisions can describe linear dependence form

$$\begin{aligned} &h_{(\tau=\infty)} = h_{max}, && \text{if } t_B < t_{min}; \\ &h_{(\tau=\infty)} = \frac{t_{max} - t_B}{t_{max} - t_{min}}, && \text{if } t_{min} \le t_B \le t_{max}; \\ &h_{(\tau=\infty)} = 0, && \text{if } t_B > t_{max}, \end{aligned} \tag{16}$$

where $h_{(\tau=\infty)} -$ rod position after transient processes;

$t_{max}, t_{min} -$ in accordance with the maximum and minimum temperature at which the valve stem reaches extreme positions, °C.

Maximum and minimum air temperature is set by turning the thermostatic valve head. To fully open the valve in ideal conditions $t_{max} - t_{min} = 2K$. In the case of this have previously configured temperature difference decreases proportionally to limit of the stroke rod.

Another important factor is the transition process in thermostatic valve head. Thus, for thermostats company "Danfoss" this time is τ_K=12–15 minutes. Given the transition processes in the head with thermostatic valve stem position can be described by the formula

$$h_{(\tau)} = h_{(\tau=\infty)} + \left(h_{(\tau=0)} - h_{(\tau=\infty)}\right)\ell^{-\frac{\tau}{\tau_K}} . \tag{17}$$

The system of equations 1-17 combines thermal and hydraulic mode of heating system in transition process and considers the work of individual thermostatic valves. The resulting system of equations thermal-hydraulic balance can be solved if you set the appropriate initial conditions.

Conclusions. The mathematical model for comprehensive consideration of the impact of various factors (both disturbing and regulating) the thermal and hydraulic modes of automated heating system with radiator thermostats. This mathematical model can be used to create computer programs and perform various research in the field of automated control thermal conditions premises.

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