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Mathematical simulation of the motion law of differential mortar pump piston intended for construction mix

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The paper is dedicated to the creation of a differential mortar pump with electromagnetic action for pumping finishing material, which is not sensitive to electric energy gaps, and which is at the same time convenient, easy to use, reliable and economical in operation. The paper presents the mathematical model of the working process dynamics of a differential mortar pump with electromagnetic action, which will allow to study common patterns of pumping processes in the pump in the whole, to solve general problems on their calculation and design, to set and solve problems of reliability control, connected with high-frequency pressure oscillations, the problems of structural optimization and optimal design of all its elements. The control system of a pumping unit with vector controlled asynchronous electric drive is proposed on the basis of the concept of inverse dynamics problems in combination with the minimization of local functionality of instantaneous energy magnitudes, which ensures high-quality pressure regulation under the conditions of parametric perturbations activity and has acceptable energy indices.

Keywords: differential mortar pump with electromagnetic action, mathematical modeling, construction mix.

Математичне моделювання закону руху поршня диференціального насоса електромагнітної дії для будівельної суміші

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Роботу присвячено створенню диференціального насоса електромагнітної дії для перекачування оздоблювального матеріалу, який не чутливий до перепадів електроенергії, зручний і простий у використанні, надійний та економічний в експлуатації. Розглянуто результати математичного моделювання нестаціонарних процесів у насосному агрегаті з однопоршневим диференціальним насосом електромагнітної дії. Проаналізовано модель, що містить рівняння руху елементів системи, котрі враховують несталість зведеного моменту інерції штокового механізму насоса, а також електромагнітні явища в електромагнітній котушці. Запропоновано математичну модель динаміки робочого процесу диференціального насоса електромагнітної дії, яка дозволить досліджувати загальні закономірності перекачувальних процесів у насосі в цілому, розв'язувати загальні задачі з їх розрахунку і проектування, ставити і розв'язувати задачі забезпечення надійності, пов'язані з високочастотними коливаннями тиску, задачі оптимізації її структури й оптимального проектування всіх її елементів. Результати розв'язання диференціальних рівнянь математичної моделі, отримані у цій статті, можуть бути рекомендовано для практичної реалізації у вигляді аналітичних залежностей при розробленні методики розрахунку для створення нових конструкцій диференціальних насосів електромагнітної дії та оцінювання їх ефективності. Запропоновано систему керування насосною установкою з векторно-керованим асинхронним електроприводом на основі концепції зворотних задач динаміки в поєднанні з мінімізацією локальних функціоналів миттєвих значень енергії, яка забезпечує якісне регулювання напору в умовах дії параметричних збурень та має задовільні енергетичні показники.

Ключові слова: диференціальний насос електромагнітної дії, математичне моделювання, будівельна суміш

Introduction

Academic specialists have been researching mortar pumps from 1950-es onward. Notwithstanding that scientific works of that period remain fundamental, they do not contain complete analysis of differential mortar pumps owith electromagnetic action, but only describe their work and design in general.

The basis for the improvement of the effectiveness of a differential mortar pump with electromagnetic action is improvement of the power efficiency required to maintain constrained oscillations and immunity to electric energy gaps.

Review of research sources and publications

Creating efficient pumping equipment is a vexed problem for pumping production sphere, as over the past 10 years mostly obsolete technologies have been used in the field of pumping equipment manufacture. At present, the crucial task is to create a differential mortar pump with electromagnetic action [7 – 14], fit for effective work, as well as to produce a mathematical model [1 – 4], which, in its turn, would describe the overall operation of a differential mortar pump with electromagnetic action. The effective work is the ability of the pump to provide the maximum possible efficiency factor, which, in its turn, depends on the interaction between the coil and the plunger [5, 6], and, as a consequence – to support high performance reliability.

Thus, differential mortar pumps with electromagnetic action are one of the most common pump types, their constructional diversity is extremely high. Obtaining the required quality of a differential pump of electromagnetic action is a current problem, which is of great importance for the development of pumping production sphere.

Problem statement

In accordance with the abovementioned, the purpose of the article is to increase the running efficiency of a differential mortar pump with electromagnetic action. In order to achieve this goal, we have solved the following task: to create a mathematical model of the influence of electromagnetic induction on the uniformity of the construction mix pumping.

Basic material and results

Let's consider the construction of a differential mortar pump with electromagnetic action for the construction mix, pictured in Fig. 1.

The mortar pump works as follows. Electric current that changes along the sinusoidal wave and induces magnetic induction onto the plunger, drawing it into the middle of the coil, enters into the coil 3.

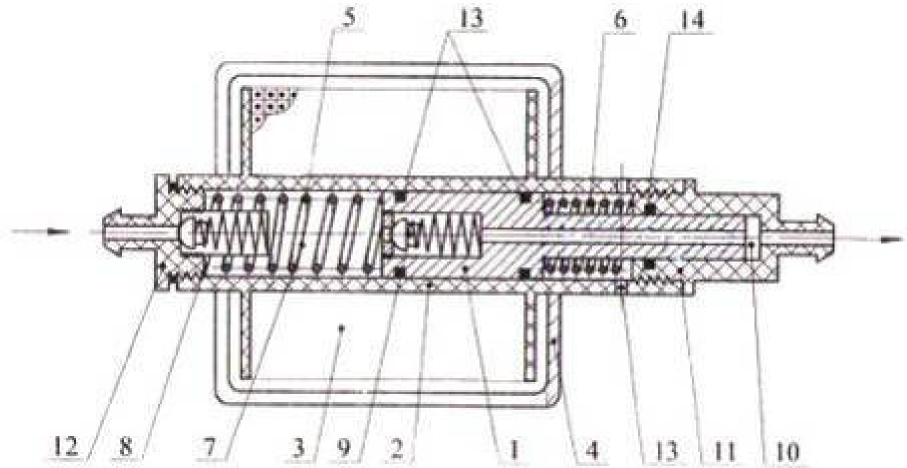

Figure 1 – The structure of the differential mortar pump with electromagnetic action for construction mix pumping:
1 – plunger, 2 – pump body; 3 – coil; 4 – coil flux guide; 5 – suction chamber; 6 – compensating spring; 7 – working spring; 8 – sniffle valve; 9 – discharge valve; 10 – compensating chamber; 11,12 – discharge fitting and suction fitting; 13,14 – lip-type seal

The first cycle of pumping. The plunger starts to move leftward, closing the sniffle valve (t1) and opening the discharge valve (t2). When the pump cavity is filled with mortar mix, the pumping process begins and the pressure in the discharge fitting starts to increase. The higher the motion speed of the plunger is, the more the pressure increases.

At the same time, the working spring 7 begins to shrink and the compensating spring 6 starts to straighten out. When the electric current in the coil falls, magnetic induction decreases and simultaneously the motion speed of the plunger declines until it stops. However, the plunger stops a little earlier before the complete shutdown of magnetic induction, when there occurs the balance moment of magnetic induction and compression force of the working spring 7. When the sinusoidal wave changes its direction, the diode in the power supply scheme cuts off its lower part, and in the second cycle, magnetic induction does not affect the plunger.

***The second cycle of pumping*.** The working spring begins to straighten out, resulting in the opposite motion when the plunger is moving. The motion speed of the plunger begins to increase. When the plunger is moving to the right, the discharge valve 9 is closed and the pumping process is reactivated. Pumping pressure increases in proportion to the increase in path velocity.

At the same time, the sniffle valve 8 is opened and the working fluid is absorbed into the working cavity of the mortar pump. With the displacement of the plunger to the right, the working spring 7 compression is weakened. Straightening of the working spring is prevented by the pumping effort of the working fluid, the effort of absorbing the fluid into the working chamber and by the compensating spring compression. With the slackening of the working spring, the motion speed of the plunger decreases and coincidently the pumping process is reduced. By the moment the plunger stops, voltage is again applied into the coil, and the pumping process is rerun.

Let’s consider mortar pump operation separately for each cycle.

The first cycle of pumping. Plunger movement to the left. Since the plunger 1 moves progressively, then, making a mathematical model of its mechanical motion, we will consider it as a point particle, the mass of which is equal to the mass *m* of the plunger.

Let's consider the motion of a point particle to the left from its full distance right position, which is pictured in Fig. 2 and we will make a computational scheme of this motion, combining the coordinate origin *Oxy* with the initial position of the point particle and pointing the axis *Ox* toward the direction of particle motion. With such a choice of reference system

$x_0 = 0$ and $\dot{x}_0 = 0$,

where x_0 – the coordinate position that determines the position of the point particle at the time $t_0 = 0$,

$\dot{x}_0$ – the projection onto the axis *Ox* of the initial velocity $\vec{v}_0$ (of course, taking into account that v_0 =0, then $\dot{x}_0 = 0$).

a) b)

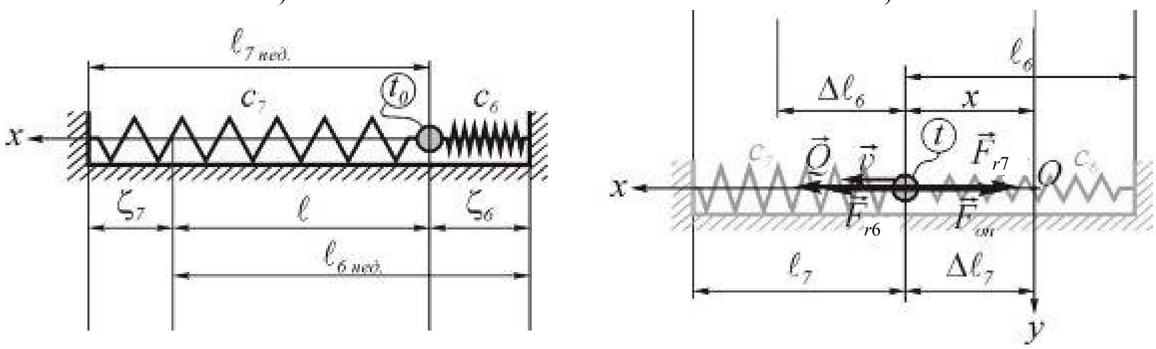

Figure 2 – Computational scheme for leftward plunger movement

Due to the fact that the motion of the point particle along the axis *Oy* is absent, then in accordance with the III-d Newton's law of motion

$\vec{G} = -\vec{N}$,

where $\vec{G} = m\vec{g}$ – gravity force, and $\vec{N}$ – normal response of the walls of the pump body.

Thus, forces $\vec{G}$ and $\vec{N}$ form a balanced system of forces $\{\vec{G}, \vec{N}\}$, which we shove aside on the basis of the corresponding axiom of statics, without breaking down the kinematic state of the point particle under consideration.

As a result, we have forces that have affect on the point particle (see figure 2, b):

– moving force $\vec{Q}$;

– elastic forces $\vec{F}_{r7}$ and $\vec{F}_{r6}$ of the compensating spring 6 and the working spring 7 accordingly;

– motion resistance force $\vec{F}_{on}$.

Let’s find out the meaning of these forces.

The module *Q* of the moving force changes sinusoidally similarly to the change of electric current by virtue of

$Q = Q_0 \cdot \sin pt$,

where Q_0 – the peak value of the moving force (needless to say, the dimension $[Q] = [Q_0] = MLT^{-2}$),

p – the cyclic frequency of the moving force, which is equal to the number of complete cycles of moving force variation per 2π seconds, and, of course, it is equal to the cyclic frequency of the electric current change.

In accordance with Hooke law, modules of elastic forces $\vec{F}_{r7}$ and $\vec{F}_{r6}$ of the compensating spring 6 and the working spring 7 determine the following dependencies

$F_{r7} = c_7 \cdot \Delta\ell_7$ and $F_{r6} = c_6 \cdot \Delta\ell_6$,

where c_7 and c_6 – the spring rate of the compensating spring 6 and the working spring 7

$\Delta\ell_7$, $\Delta\ell_6$ – their deformation in the position of the point particle at any given time t, pictured in Fig. 2.

The physical content of the coefficients c_7 and c_6 – the elastic force value of each spring under deformation, set to unity, and their dimensions are the ratio of the force dimension MLT^{-2} to the length dimension L, that is

$$[c_7]=[c_6]=\frac{MLT^{-2}}{L}=M\cdot T^{-2}.$$

From Fig. 2, b it is obvious that

$$\Delta\ell_7 = x,$$

$$\Delta\ell_6=\ell_{6нед.}-\ell_6=\ell_{6нед.}-(x+\varsigma_6)=\ell_{6нед.}-x-\varsigma_6,$$

where x – the coordinate, which determines the position of the point particle at the time t,

$\ell_{6нед.}$ – the length of the undeformed spring 6 ℓ_6 – the length of this spring at the time t

ς_6 – the length of this spring in the initial position of the point particle (at the time $t_0=0$).

Taking into account the set values $\Delta\ell_7$ and $\Delta\ell_6$, we will get

$$F_{r7}=c_7\cdot x,$$

$$F_{r6}=c_6\cdot(\ell_{6нед.}-x-\varsigma_6).$$

Since the plunger 1 in the stationary medium – in mortar mix, which fills mortar pump's working cavity, performs progressive motion at low speed, then the motion resistance force will be in opposition to the direction of the plunger velocity vector $\vec{v}$ and in vector form it can be written as:

$$\vec{F}_{on}=-F_{on}\cdot\frac{\vec{v}}{v},$$

where F_{on} – the absolute value (module) of this force;
v – plunger velocity module 1.

With the help of dimensional method [1] we will define the module value F_{on} of mortar mix resistance force. In initial approximation we will assume that this force, which dimension is $[F_{on}] = MLT^{-2}$, is determined by the following parameters, which, of course, are physical values:
v – the plunger motion speed, the dimension of which is $[v] = LT^{-1}$;
S – the area of the plunger pressure on the operating environment (mortar mix), the dimension of which is $[S] = L^2$;
μ – the absolute viscosity coefficient, the dimension of which is $[\mu] = ML^{-1}T^{1}$.

According to [2], we will seek functional relationship $F_{on}=f(v,S,\mu)$ in the form:

$$F_{on}=k\cdot v^a\cdot S^b\cdot\mu^c, \tag{1}$$

where k – some nondimensional coefficient (viz $[k]=1$), which cannot be determined by use of dimensional analysis.

According to the theory of dimensional analysis, between dimensions $[F_{on}]$, $[v]$, $[S]$ and $[\mu]$ there must be functional connection similar to the association between physical values F_{on}, v, S and μ, which is determined by formula (1). From this, we have that

$$[F_{on}]=[k]\cdot[v]^a\cdot[S]^b\cdot[\mu]^c,$$

or, taking into account the above-mentioned dimensions,

$$MLT^{-2}=1\cdot\left(LT^{-1}\right)^a\cdot\left(L^2\right)^b\cdot\left(ML^{-1}T^{-1}\right)^c.$$

Having completed the obvious transformations of the right side, we will obtain

$$MLT^{-2}=L^{a+2b-c}\cdot T^{-a-c}\cdot M^c.$$

Since the mathematically obtained dependence can be fulfilled only if power coefficients of the corresponding multiplicators are equal, then, making the specified indicators equal, we will obtain the system of algebraic equations:

Having solved this system of algebraic equations, we will find that

$$c=1,\ a=2-c=2-1=1$$

and

$$b=\frac{1-a+c}{2}=\frac{1-1+1}{2}=\frac{1}{2}.$$

Then the desired dependence (1) will have the form

$$F_{on}=k\cdot v^1\cdot S^{\frac{1}{2}}\cdot\mu^1$$

or

$$F_{on}=k\cdot V\cdot\mu\cdot\sqrt{S}.$$

From Fig. 1 it is clear that when the plunger 1 moves to the left, the area S of its pressure on the operating environment (mortar mix) will be determined by the formula

$$S=\frac{\pi\cdot d_1^2}{4},$$

where d_1 – the diameter of the plunger in the working chamber (see Fig. 3).

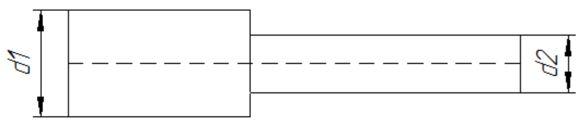

Figure 3 – Diagrammatic representation of the plunger 1

If we take $d_{om.}=\gamma\cdot d_1$, where γ – a certain "diameter reduction factor" (of course, that $\gamma<1$), then

$$S=\frac{\pi}{4}\cdot\left(d_1^2-\gamma^2\cdot d_1^2\right)=\frac{\pi\cdot d_1^2}{4}\cdot\left(1-\gamma^2\right).$$

Then finally we will get that

$$F_{on} = k\cdot v\cdot \mu \cdot \sqrt{\frac{\pi\cdot d_1^2}{4}\cdot\left(1-\gamma^2\right)} =$$

$$= \frac{k\cdot v\cdot \mu\cdot d_1\cdot\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{2}$$

or

$$F_{on} = k\cdot\frac{\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{2}\cdot\mu\cdot d_1\cdot v .$$

In the general case, the coefficient k in the resistance force formula for the resistance F_{on} of the medium is in the functional relationship to the Reynolds number (Reynolds criterion) *Re* and to the Froude number F_{on} that is

$k = f(Re, Fr)$.

According to [3], «the mathematical relation $k = f(Re, Fr)$ is complex and its extremely difficult to obtain it theoretically. It is common practice to use experimentally obtained values of the coefficient k ». But in the case under consideration, when the distance and motion speed of the point particle are insignificant, we will neglect the dependence $k = f(Re, Fr)$, assuming that $k = const$.

Following the algorithm of solving the inverse primal dynamic problem of the point particle [4], we will compose the differential equation of motion of the point particle under the question.

We will record Newton's second law of motion (second principle of dynamics) in the projection on the axis *Ox*.

From the computational scheme (see Fig. 2, b) it is obvious that

$$\sum_{i=1}^{\lambda} F_{ix} = Q + F_{r6} - F_{r7} - F_{on} \qquad (2)$$

or, taking into consideration the found force values,

$$\sum_{i=1}^{\lambda} F_{ix} = Q_0\cdot\sin\frac{\pi\cdot t}{\tau} + c_6\cdot\left(\ell_{6\,нед.} - x - \varsigma_6\right) -$$
$$- c_7\cdot x - k\cdot\frac{\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{2}\cdot\mu\cdot d_1\cdot v.$$

Then

$$\sum_{i=1}^{\lambda} F_{ix} = Q_0\cdot\sin\frac{\pi\cdot t}{\tau} + c_6\cdot\left(\ell_{6\,нед.} - x - \varsigma_6\right) -$$
$$- c_7\cdot x - k\cdot\frac{\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{2}\cdot\mu\cdot d_1\cdot v =$$
$$= Q_0\cdot\sin\frac{\pi\cdot t}{\tau} - c_6\cdot x + c_6\cdot\left(\ell_{6\,нед.} - \varsigma_6\right) -$$
$$- c_7\cdot x - k\cdot\frac{\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{2}\cdot\mu\cdot d_1\cdot v =$$
$$= Q_0\cdot\sin\frac{\pi\cdot t}{\tau} - \left(c_6 + c_7\right)\cdot x + c_6\cdot\left(\ell_{6\,нед.} - \varsigma_6\right) -$$
$$- k\cdot\frac{\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{2}\cdot\mu\cdot d_1\cdot v.$$

Substituting the last expression and the value $a_x = \frac{dv}{dt}$ into formula (2), we will obtain the differential equation of point particle motion in the form

$$m\cdot\frac{dv}{dt} = Q_0\cdot\sin\frac{\pi\cdot t}{\tau} - \left(c_6 + c_7\right)\cdot x +$$
$$+ c_6\cdot\left(\ell_{6\,нед.} - \varsigma_6\right) - k\cdot\frac{\sqrt{\pi\left(1-\gamma^2\right)}}{2}\cdot\mu\cdot d_1\cdot v ,$$

and, dividing both parts of it into m and carrying out legitimate transformations, we will get

$$\frac{dv}{dt} = \frac{Q_0}{m}\cdot\sin\frac{\pi\cdot t}{\tau} - \frac{\left(c_6 + c_7\right)}{m}\cdot x +$$
$$+ \frac{c_6\cdot\left(\ell_{6\,нед.} - \varsigma_6\right)}{m} - k\cdot\frac{\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{2\cdot m}\cdot\mu\cdot d_1\cdot v$$

and

$$\frac{dv}{dt} + k\cdot\frac{\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{2\cdot m}\cdot\mu\cdot d_1\cdot v + \frac{c_6 + c_7}{m}\cdot x =$$
$$= \frac{Q_0}{m}\cdot\sin\frac{\pi\cdot t}{\tau} + \frac{c_6\cdot\left(\ell_{6\,нед.} - \varsigma_6\right)}{m}. \qquad (3)$$

The deduced equation (3) is the differential equation of plunger movement 1 to the left in the canonical form or the mathematical model of this mechanical motion.

If we introduce the designations, traditional for the theory of oscillations [5]

$$k\cdot\frac{\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{2\cdot m}\cdot\mu\cdot d_1 = 2\cdot n_l ,$$

$$\frac{c_6 + c_7}{m} = k^2 , \qquad \frac{Q_0}{m} = h$$

and assume by convention that

$$\frac{c_6\cdot\left(\ell_{6\,нед.} - \varsigma_6\right)}{m} = C_1 = const ,$$

then the differential equation of plunger movement 1 to the left can be given more compact form

$$\frac{dv}{dt} + 2\cdot n_l\cdot v + k^2\cdot x = h\cdot\sin\frac{\pi\cdot t}{\tau} + C_1 .$$

Let's determine dimensions of physical values n_l, k, h and C_1. In accordance with the foregoing formula

$$n_l = k\cdot\frac{\sqrt{\pi\cdot\left(1-\gamma^2\right)}}{4\cdot m}\cdot\mu\cdot d_1 ,$$

$$k = \sqrt{\frac{c_6 + c_7}{m}} .$$

Then

$$\left[n_{лів.}\right] = \left[k\right]\cdot\frac{1}{\left[m\right]}\cdot\left[\mu\right]\cdot\left[d_1\right] = 1\cdot\frac{1}{M}\cdot ML^{-1}T^{-1}\cdot L =$$
$$= \frac{M\cdot L}{M\cdot L\cdot T} = \frac{1}{T} = T^{-1} ,$$

$$[k]=\sqrt{\frac{[c_6]+[c_7]}{[m]}}=\sqrt{\frac{M\cdot T^{-2}+M\cdot T^{-2}}{M}}= \\ =\sqrt{\frac{M\cdot T^{-2}}{M}}=\sqrt{T^{-2}}=T^{-1},$$

$$[h]=\frac{[Q_0]}{[m]}=\frac{MLT^{-2}}{M}=L\cdot T^{-2},$$

$$[C_1]=\frac{[c_6]\cdot([\ell_{6\,нед.}]-[\varsigma_6])}{[m]}=\frac{MT^{-2}\cdot(L-L)}{M}= \\ =\frac{MT^{-2}\cdot L}{M}=L\cdot T^{-2}.$$

Note that the values k and n_l have the same dimensions, which allows, if necessary, to compare these values. Both values h and C_1, as it must be, have identical dimensions.

In the theory of oscillations according to [5] physical values n_l, k and h have the following mechanical intensions:

n_l – the attenuation coefficient, which characterizes the resistance of medium at low motion speed of the point particle;

k – cyclic/circular (radian) frequency of eigenvibrations (free oscillations) of the point particle on the spring with stiffness coefficient $c_{ecv} = c_6 + c_7$;

h – the largest value of the summand, which determines the maximum acceleration of the motion of the point particle $a_{max} = h + C_1$ under investigation.

Let's solve the equation (4) and construct acceleration profile of the plunger to the left using the free mathematical program "SMath Studio" taking into account the initial conditions:

$$\begin{cases} v_{(0)}=0; \\ P_{(0)}=P_{\min}; \\ x_{(0)}=0. \end{cases}$$

The constructed graph is shown in Fig. 4. The received peak speed of the plunger, taking into account given geometrical dimensions of mortar pump component parts, proceeding from the diameter of the working chamber section (plunger diameter) 25 mm at the productivity of 0,25 m^3/h is 8,63 m/s.

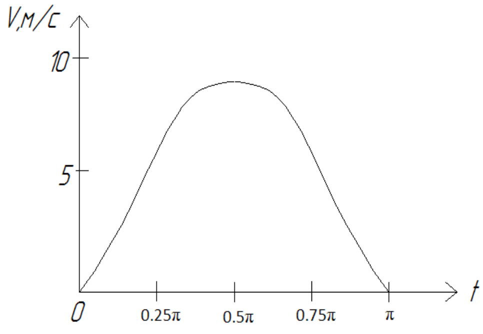

Figure 4 – The graph of plunger speed variation in time when it moves to the left

The second cycle of pumping. Plunger movement to the right. Now let us consider the motion of the point particle to the right from its full distance left position, which is pictured in Figure 2, a, and in Fig. 2, b and we will make a computational scheme of this motion, by choosing the system of coordinates Oxy from the conditions similar to those of the point particle motion on the left. When choosing such a frame of reference, the initial conditions of motion will be:

$$x_0=0 \text{ and } \dot{x}_0=0,$$

and forces $\vec{F}_{r7}$, $\vec{F}_{r6}$ and $\vec{F}_{on}$ .affect the point particle (see Fig. 4, b)

In accordance with Hooke law, $F_{r7}=c_7\cdot\Delta\ell_7$ and $F_{r6}=c_6\cdot\Delta\ell_6$,

From Figure 2, b it is obvious that

$$\Delta\ell_6=x,$$

$$\Delta\ell_7=\ell_{7\,нед.}-\ell_7=\ell_{7\,нед.}-(\varsigma_7+x)= \\ =\ell_{7\,нед.}-\varsigma_7-x,$$

where x – the coordinate that determines the point particle position at the time t, $\ell_{7\,нед.}$ – the length of the undeformed spring 7, ℓ_7 – the length of this spring at the time t and ς_7 – the length of this spring in the initial position of the point particle (at the time $t_0=0$).

Taking into account the set values $\Delta\ell_7$ and $\Delta\ell_6$, we will get

$$F_{r7}=c_7\cdot(\ell_{7\,нед.}-\varsigma_7-x),$$

$$F_{r6}=c_6\cdot x.$$

The module of the resistance force F_{on} of the mortar mix to the point particle motion is again determined from the formula

$$F_{on}=k\cdot v\cdot\mu\cdot\sqrt{S},$$

and from Fig. 3 it is clear that when the plunger 1 moves to the right, its area of pressure on the working medium (solution) will be as follows

$$S=\frac{\pi\cdot d_2^2}{4}.$$

Then in this case

$$F_{on}=k\cdot v\cdot\mu\cdot\sqrt{\frac{\pi\cdot d_2^2}{4}}=\frac{k\cdot v\cdot\mu\cdot d_2\cdot\sqrt{\pi}}{2}$$

or

$$F_{on}=k\cdot\frac{\sqrt{\pi}}{2}\cdot\mu\cdot d_2\cdot v.$$

From the computational scheme (see Fig. 2, b), we can define, that in this case

$$\sum_{i=1}^{\lambda} F_{ix} = F_{r7} - F_{r6} - F_{on}\,, \tag{4}$$

or, taking into account the set force values,

$$\sum_{i=1}^{\lambda} F_{ix} = c_7 \cdot \left(\ell_{7\,нед.} - \varsigma_7 - x\right) - c_6 \cdot x - k \cdot \frac{\sqrt{\pi}}{2} \cdot \mu \cdot d_2 \cdot v =$$

$$= c_7 \cdot \left(\ell_{7\,нед.} - \varsigma_7\right) - c_7 \cdot x - c_6 \cdot x - k \cdot \frac{\sqrt{\pi}}{2} \cdot \mu \cdot d_2 \cdot v =$$

$$= c_7 \cdot \left(\ell_{7\,нед.} - \varsigma_7\right) - \left(c_7 + c_6\right) \cdot x - k \cdot \frac{\sqrt{\pi}}{2} \cdot \mu \cdot d_2 \cdot v\,.$$

Substituting the last expression and the value $a_x = \frac{dv}{dt}$ into formula (4), we will obtain the differential equation of point particle motion in the form

$$m \cdot \frac{dv}{dt} = c_7 \cdot \left(\ell_{7\,нед.} - \varsigma_7\right) - \left(c_7 + c_6\right) \cdot x - k \cdot \frac{\sqrt{\pi}}{2} \cdot \mu \cdot d_2 \cdot v\,,$$

and, dividing both parts of it into m and carrying out legitimate transformations, we will get

$$\frac{dv}{dt} = \frac{c_7 \cdot \left(\ell_{7\,нед.} - \varsigma_7\right)}{m} - \frac{\left(c_7 + c_6\right)}{m} \cdot x - k \cdot \frac{\sqrt{\pi}}{2 \cdot m} \cdot \mu \cdot d_2 \cdot v$$

and

$$\frac{dv}{dt} + k \cdot \frac{\sqrt{\pi}}{2 \cdot m} \cdot \mu \cdot d_2 \cdot v + \frac{c_7 + c_6}{m} \cdot x = \frac{c_7 \cdot \left(\ell_{7\,нед.} - \varsigma_7\right)}{m}\,. \tag{5}$$

The deduced equation (5) is the differential equation of plunger movement 1 to the right in the canonical form or the mathematical model of this mechanical motion.

Again, if we introduce the designations, traditional for the theory of oscillations, we will get:

$$k \cdot \frac{\sqrt{\pi}}{2 \cdot m} \cdot \mu \cdot d_2 = 2 \cdot n_r\ ;$$

$$\frac{c_7 + c_6}{m} = k^2$$

and assume by convention that

$$\frac{c_7 \cdot \left(\ell_{7\,нед.} - \varsigma_7\right)}{m} = C_2 = const\ ,$$

then the above-obtained differential equation of plunger movement 1 to the right can be given the following form

$$\frac{dv}{dt} + 2 \cdot n_{пр.} \cdot v + k^2 \cdot x = C_2\,,$$

in which the dimensions of physical values n_r, k and C_2 as well as their mechanical intensions, are undoubtedly similar to the foregoing.

Let's solve the equation (2) and construct acceleration profile of the plunger to the right at the same axis of reference with acceleration profile of the plunger to the left using the free mathematical program "SMath Studio" taking into account the initial conditions:

$$\begin{cases} v_{(0)} = 0; \\ P_{(0)} = P_{\min};, \\ x_{(0)} = 0. \end{cases}$$

The constructed graph is shown in Fig. 5 and displays the graph of temporal variations for the plunger speed in the complete cycle. The received peak speed of the plunger movement to the right, taking into account given geometrical dimensions of mortar pump component parts, proceeding from the diameter of the working chamber section (plunger diameter) 25 mm at the productivity of 0,25 m^3/h is 9,33 m/s.

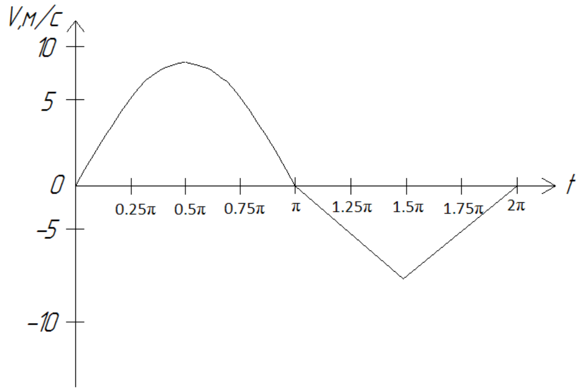

Figure 5 – The graph of temporal variations for the plunger speed in the complete cycle of plunger movement

Conclusions

1. We have obtained mathematical models of differential equations, which reflect velocity history (variations in time of the velocity) of a differential pump piston of a mortar pump used for pumping of construction mixes in the complete cycle of its movement.

2. The analysis of the obtained mathematical models allows to optimize the geometric dimensions of mortar pump component parts, including the geometric dimensions of the springs in order to ensure the mechanical energy conservation during pumping.

3. The graph of temporal variations for the plunger speed in the complete cycle of plunger movement allows us to simulate the required plunger productivity when pumping mortar mixes.

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